

APPLIED MATHEMATICS (241)

Date: 12/12/2022

MODEL EXAMINATION (2022-23) Grad

Grade: XII

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions

- 1. This question paper contains five sections A, B, C, D and E
- 2. Section-A carries 20 marks weightage, Section-B carries 10 marks weightage, Section-C carries 18 marks weightage, Section-D carries 20 marks weightage and Section-E carries 3 case-based with total weightage of 12 marks

Section-A

3. It comprises of 20 MCQs if 1 mark each

Section-B

- 4. It comprises of **5 VSA** type questions of **2 marks** each <u>Section-C</u>
- 5. It comprises of **6 SA** type questions of **3 marks** each <u>Section-D</u>
- 6. It comprises of **4 LA** type questions of **5 marks** each **Section-E**
- 7. It has **3 case studies**. Each case study comprises of 3 case-based questions.
- 8. Internal choice is provided in 2 questions in **Section-B**, 2 questions in **Section-C**, 2 questions in **Section-D**. You have to attempt only one of the alternatives in all such questions.

SECTION-A

(All questions are compulsory, No internal choice is provided in this section)

- 1. Find the least positive value of x such that $71 \equiv x \pmod{8}$. a) 7 b) 8 c) 9 d) 6
- 2. Let p > 0 and q < 0 and $p, q \in Z$, then choose the correct inequality from the given below options to complete the statement, p + q p q

 $a) > b) < c) \le d) \ge$

- 3. For the purpose of t-test of significance, a random sample of size (n) 48 is drawn from a normal population, then the degree of freedom is
 - a) 40 b) 24 c) 49 d) 47
- 4. Standard deviation of a sample from a population is called
 - a) standard error b) degree of freedom

c) level of significance d) statistic

5. A boat can row upstream at 10 km/h and downstream at 18 km/h. Find the speed of boat in still water.

a) 14 km/h b) 15 km/h c) 16 km/h d) 17 km/h

- 6. Find the present value of a sequence of payments of ₹1000 made at the end of every 6 months and continuing forever, if money is worth 8% per annum compounded semi-annually.
 a) 28500 b) 27000 c) 25000 d) 24800
- 7. In the given figure, the area bounded by the curve x = f(y), y axis and abscissa y = a and y = b is equal to



8. A machine costing ₹ 50000 has a useful life of 4 years. The estimated scrap value is ₹ 10000. Using straight line method find the annual depreciation.

a) 1200 b) 11000 c) 10000 d) 15000

9. The probability distribution of a discrete random variable X is given below.



10. The rise in prices before Diwali is an example ofa) Seasonal trendb) cyclical trendc) long term trend

11. A person has an initial investment of ₹50000 in an investment plan. After 2 years it has grown to ₹60000. Find his rate of return.
a) 20%
b) 25%
c) 21%
d) 23%

d) irregular trend

12. In a school, a random sample of 145 students is taken to check whether a student's average calory in take is 1500 or not. The collected data of average calories intake of sample students is presented in a frequency distribution, which is called

a) Statistics b) sampling distribution c) parameter d) population sampling 13. For the given five values, 15, 24, 18, 33, 42, the 3 years moving average are

- a) 19, 22, 33 b) 19, 25, 31 c) 19, 30, 31 d) 19, 25, 33
- 14. In what ratio must a shopkeeper mix two types of oranges worth ₹ 30 per kg and ₹ 45 per kg respectively so as to get a mixture at ₹ 40 per kg.
 - a) 1/5 b) $\frac{1}{2}$ c) 3/5 d) 1/5
- 15. The area of the region bounded by curve $x^2 = y$, the line x = 3 and the x axis is, a) 9 sq. units b) 6 sq. units c) 18 sq. units d) 10 sq. units
- 16. A statement made about a population parameter for testing purpose is called.....Statisticb) parameterc) hypothesisd) level of significance

17. Given that the total cost function for x units of a commodity is $C(x) = \frac{x^3}{3} + 3x^2 - 7x + 16$, then the marginal cost is,

a) $x^2 + 6x - 7$ b) $x^4 + 39x^2 - 7x + 16$ c) $x^3 - 6x + 16$ d) $x^3 + 6x + 16$

18. Find the product of order and degree of the differential equation $\left[\frac{dy}{dx}\right]^5 + 3xy\left[\frac{d^3y}{dx^3}\right]^2 + y\left[\frac{d^2y}{dx^2}\right]^4 = 0.$

a) 5 b) 6 c) 8 d) 5

For questions 19 and 20, two statements are given-one labeled Assertion(A) and the other labeled Reason(R). Select the correct answer to these questions from the codes (i), (ii), (iii), and (iv) as given below.

(i) Both A and R are true and R is the correct explanation of the assertion

(ii) Both A and R are true but R is not the correct explanation of the assertion

(iii) A is true, but R is false

(iv) A is false, but R is true

19. Assertion (A): The slope of normal to the curve $x^2 + 2y + y^2 = 0$ at (-1, 2) is -3 Reason (R) : The slope of tangent to the curve $x^2 + 2y + y^2 = 0$ at (-1, 2) is 1/3. a) (i) b) (ii) c) (iii) d) (iv)

20. Assertion (A): If nominal rate of interest is 12.5% and the inflation is 2%, then the effective rate of interest is 10.5%

Reason (R) : If the interest is calculated only at the end of an year, then the effective rate of interest is same as the nominal rate of interest.

a) (i) b) (ii) c) (iii) d) (iv)

SECTION B

(All questions are compulsory. In case of internal choice, attempt any one question only)

21. A die is thrown 6 times. If "getting an odd number" is a success, What is the probability of getting at most 5 successes?

22. Find the value of
$$x - y + z$$
 if $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$.
OR
Find the value of $a + 3b - c$ if $A = \begin{bmatrix} 0 & -1 & 28 \\ a - 8 & 0 & 3b \\ -c + 2 & 2 & 0 \end{bmatrix}$ is a skew symmetric matrix.

- 23. A retired person wants to invest an amount of ₹20000. His broker recommends investing in two types of bonds A and B. Bond A yielding 8% return on the amount invested and bond B yielding 7% return on the amount invested. After some consideration, he decides to invest at least ₹5000 in bond A and no more than ₹8000 in bond B. He also wants to invest at least as much in bond A as in bond B. Formulate as LPP to maximize his return on investments.
- 24. A pump can fill a tank with water in 3 hours, because of a leakage in the tank it took $3\frac{1}{3}$ hours to fill the tank. How much time will it take for the leakage to drain all the water of the full tank?

A can run 22.5 m while B runs 25 m in the same time. In a 1000 m race, find how much distance B beats A?

25. Find the effective rate which is equivalent to a stated rate of 6% compounded semiannually

SECTION C

(All questions are compulsory. In case of internal choice, attempt any one question only)

- 26. Find the intervals in which the function $f(x) = x^4 \frac{4x^3}{3}$ is strictly increasing and strictly decreasing.
- 27. The population of a town grows at the rate of 10% per year. Using differential equation, find how long will it take for the population to grow 4 times.
- 28. A company borrowed ₹60,000 for renovation. The company plans to set up a sinking fund that will pay back the loan at the end of 5 years. Assuming a rate of 10% compounded semiannually, and the sinking fund of the ordinary annuity. Given that $(1.05)^{10} = 1.06288$
- 29. Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats, amd plates from recycle material at a cost of ₹25, ₹100 and ₹50 each. The number of articles sold are given below

SchoolArticles	А	В	С
Fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Calculate the fund collected by Schools A,B and C by matrix Algebra. 30. Evaluate $\int e^{3logx} (x^4 + 1)^{-1} dx$.

OR

Evaluate $\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1} dx$.

31. The demand function for a commodity is $p = 20e^{\frac{-x}{10}}$. Find the consumer's surplus at equilibrium price p = 2. (*Given* $\log_{10} e = 0.4343$).

OR

The supply function for a commodity is p = 4 + x. Determine producer's surplus if 12 units of goods are sold.

SECTION D

(All questions are compulsory. In case of internal choice, attempt any one question only)

32. Consider the following hypothesis test

$$H_0: \mu \le 12$$

 $H_a: \mu > 12$

A sample of 25 provided a sample mean $\overline{x} = 14$ and a sample standard deviation S=4.32.

(i) Compute the value of the test statistic.

(ii) Use the t-distribution table to compute a range for the p-value

(iii) At $\propto = 0.05$, what is your conclusion

(iv) What is the rejection rule using the critical value? What is your conclusion?

33. The average number, in lakhs, of working days lost in strikes during each year of the period (1981-1990) was as under.

1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three-yearly moving average and draw the moving average graph.

34. In a bank, principal increased continuously at the rate of 5% per year. In how many years ₹1000 double itself?

OR

The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units, find the radius of the balloon after t seconds.

35. A furniture dealer deals in only items-chairs and tables. He has ₹50000 to invest and a space to store at most 5 items. A chair costs him ₹1000 and a table costs him ₹2000. The trader earns a profit of ₹150 and ₹250 on a chair and a table respectively. Formulate the above problem as an LPP to maximize the profit and solve it graphically.

OR

One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 kg of fat, assuming that there is no shortage of the other ingredients used in making the cake. Make it an L.P.P and solve it graphically.

<u>SECTION E</u>

(All questions are compulsory. In case of internal choice, attempt any one question only) **CASE STUDY-I**

36. A pipe is connected to a tank or cistern. It is used to fill or empty the cistern. The amount of work done by a pipe is a part of the tank filled or emptied in unit time. Three pipes A, B and C are connected to a tank. A and B fill the tank in 6 hours and 8 hours respectively when operated independently. Pipe C empty the full tank in 12 hours when opened alone. Based on the above information answer the following questions (i) If pipes A and C are opened together, then the tank can be filled in a) 12 hours b) 16 hours c) 20 hours d) 24 hours (ii)If pipes B and C opened together, then the tank can be filled in a) 12 hours b) 16 hours c) 20 hours d) 24 hours (iii)If all pipes A, B and C are opened together, then the tank can be filled in a) 4.2 hours b) 4.6 hours c) 4.8 hours d) 5 hours **CASE STUDY-II** 37. In year 2000 Mr. Talwar took a home loan of ₹3000000 from State Bank of India at 7.5% p.a compounded monthly for 20 years. Based on the above information, answer the following questions (i) The equated monthly installment paid by Mr. Talwar was: a) ₹24167.82 b) ₹35288.82 c) ₹37839.85 d) ₹39437.75 (ii)Interest paid by Mr. Talwar in 150th payment was: a) ₹20132.84 b) ₹15712.67 c) ₹10458.69 d) ₹9532.76

	(iii) Principal paid by	Mr. Talwar in 150 th	¹ payment was:	
	a) ₹4034.98	b) ₹8455.15	c) ₹14635.06	d) ₹13709.13
38.	CASE STUDY-III			



The graph given above showing the demand and supply curves of a commodity, which are linear. When the price of the commodity was ₹4000 per unit, Gulati and Sons sold 20 units every month and when price dropped to ₹1000 per unit Gulati and sons sold 120 units per month. When price was ₹4000 per unit 200 units were available per month for sale and when price was ₹1000 per unit only 50 units remained.

Based on the above information, answer the following questions

(i) The demand function	on is					
a) $D(x) = 30x - 4$	·600	b) $D(x) = 4600 - 30x$				
c) $D(x) = 35x + 4500$		d) $D(x) = 4500 - 35x$				
(ii) The equilibrium point is						
a) (100, 1500)	b) (83, 1760)	c) (92, 1840)	d) (95, 1850)			
(iii) The consumer surplus is						
a) ₹125460	b) ₹126960	c) ₹135500	d) ₹ 135520			
